

# Standard Test Method for Measuring Relative Complex Permittivity and Relative Magnetic Permeability of Solid Materials at Microwave Frequencies Using Waveguide<sup>1</sup>

This standard is issued under the fixed designation D5568; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

## 1. Scope\*

1.1 This test method covers a procedure for determining relative complex permittivity (relative dielectric constant and loss) and relative magnetic permeability of isotropic, reciprocal (non-gyromagnetic) solid materials. If the material is nonmagnetic, it is acceptable to use this procedure to measure permittivity only.

1.2 This measurement method is valid over a frequency range of approximately 100 MHz to over 20 GHz. These limits are not exact and depend on the size of the specimen, the size of rectangular waveguide transmission line used as a specimen holder, and on the applicable frequency range of the network analyzer used to make measurements. The practical lower and upper frequencies are limited by specimen dimension requirements (large specimens at low frequencies and small specimens at high frequencies). Being a non-resonant method, the selection of any number of discrete measurement frequencies in a measurement band would be suitable. Use of multiple rectangular waveguide transmission line sizes are required to cover this entire frequency range (100 MHz to 20 GHz). This test method can also be generally applied to circular waveguide test fixtures. The rectangular waveguide fixture is preferred over coaxial fixtures when samples have in-plane anisotropy or are difficult to manufacture precisely.

1.3 The values stated in SI units are to be regarded as the standard. The values given in parentheses are in English units. The equations shown here assume an  $e^{+j\omega t}$  harmonic time convention.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

## 2. Referenced Documents

2.1 *ASTM Standards:*<sup>2</sup>

**D1711** Terminology Relating to Electrical Insulation

## 3. Terminology

3.1 For other definitions used in this test method, refer to Terminology **D1711**.

3.2 *Definitions:*

3.2.1 *relative complex permittivity (relative complex dielectric constant),  $\epsilon_r^*$ ,  $n$* —the proportionality factor that relates the electric field to the electric flux density, and which depends on intrinsic material properties such as molecular polarizability, charge mobility, etc.:

$$\epsilon_r^* = \epsilon_r' - j\epsilon_r'' = \frac{\vec{D}}{\epsilon_0 \vec{E}} \quad (1)$$

where:

$\epsilon_0$  = the permittivity of free space,

$\vec{D}$  = the electric flux density vector, and

$\vec{E}$  = the electric field vector.

3.2.1.1 *Discussion*—In common usage the word “relative” is frequently dropped. The real part of complex relative

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<sup>2</sup> For referenced ASTM standards, visit the ASTM website, [www.astm.org](http://www.astm.org), or contact ASTM Customer Service at [service@astm.org](mailto:service@astm.org). For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

permittivity ( $\epsilon_r'$ ) is often referred to as simply relative permittivity, permittivity or dielectric constant. The imaginary part of complex relative permittivity ( $\epsilon_r''$ ) is often referred to as the loss factor. In anisotropic media, permittivity is described by a three dimensional tensor.

3.2.1.2 *Discussion*—For the purposes of this test method, the media is considered to be isotropic, and therefore permittivity is a single complex number at each frequency.

3.2.2 *relative complex permeability*,  $\mu_r^*$ ,  $n$ —the proportionality factor that relates the magnetic flux density to the magnetic field, and which depends on intrinsic material properties such as magnetic moment, domain magnetization, etc.:

$$\mu_r^* = \mu_r' - j\mu_r'' = \frac{\vec{B}}{\mu_0 \vec{H}} \quad (2)$$

where:

$\mu_0$  = the permeability of free space,

$\vec{B}$  = the magnetic flux density vector, and

$\vec{H}$  = the magnetic field vector.

3.2.2.1 *Discussion*—In common usage the word “relative” is frequently dropped. The real part of complex relative permeability ( $\mu_r'$ ) is often referred to as relative permeability or simply permeability. The imaginary part of complex relative permeability ( $\mu_r''$ ) is often referred to as the magnetic loss factor. In anisotropic media, permeability is described by a three dimensional tensor.

3.2.2.2 *Discussion*—For the purposes of this test method, the media is considered to be isotropic, and therefore permeability is a single complex number at each frequency.

### 3.3 Definitions of Terms Specific to This Standard:

3.3.1 A list of symbols specific to this test method is given in **Annex A1**.

3.3.2 *calibration*,  $n$ —a procedure for connecting characterized standard devices to the test ports of a network analyzer to characterize the measurement system’s systematic errors. The effects of the systematic errors are then mathematically removed from the indicated measurements. The calibration also establishes the mathematical reference plane for the measurement test ports.

3.3.2.1 *Discussion*—Modern network analyzers have this capability built in. There are a variety of calibration kits that can be used depending on the type of test port. The models used to predict the measurement response of the calibration devices depends on the type of calibration kit. Most calibration kits come with media that can be used to load the definitions of the calibration devices into the network analyzer. Calibration kit definitions loaded into the network analyzer must match the devices used to calibrate. Since both transmission and reflection measurements are used in this standard, a two-port calibration is required.

3.3.3 *network analyzer*,  $n$ —a system that measures the two-port transmission and one-port reflection characteristics of a multiport system in its linear range and at a common input and output frequency.

3.3.3.1 *Discussion*—For the purposes of this standard, this description includes only those systems that have a synthesized signal generator, and that measure the complex scattering

parameters (both magnitude and phase) in the forward and reverse directions of a two-port network ( $S_{11}$ ,  $S_{21}$ ,  $S_{12}$ ,  $S_{22}$ ).

3.3.4 *scattering parameter (S-parameter)*,  $S_{ij}$ ,  $n$ —a complex number consisting of either the reflection or transmission coefficient of a component at a specified set of input and output reference planes with an incident signal on only a single port.

3.3.4.1 *Discussion*—As most commonly used, these coefficients represent the quotient of the complex electric field strength (or voltage) of a reflected or transmitted wave divided by that of an incident wave. The subscripts  $i$  and  $j$  of a typical coefficient  $S_{ij}$  refer to the output and input ports, respectively. For example, the forward transmission coefficient  $S_{21}$  is the ratio of the transmitted wave voltage at Reference Plane 2 (Port 2) divided by the incident wave voltage measured at Reference Plane 1 (Port 1). Similarly, the Port 1 reflection coefficient  $S_{11}$  is the ratio of the Port 1 reflected wave voltage divided by the Port 1 incident wave voltage at reference plane 1 (Port 1).

3.3.5 *transverse electric (TE<sub>mn</sub>) wave*,  $n$ —an electromagnetic wave in which the electric field is everywhere perpendicular to the direction of propagation.

3.3.5.1 *Discussion*—The index  $m$  is the number of half-period variations of the field along the waveguide’s larger transverse dimension, and  $n$  is the number of half-period variations of the field along the waveguide’s smaller transverse dimension. The dominant wave in a rectangular waveguide is  $TE_{10}$ . The electric field lines of the  $TE_{10}$  mode are parallel to the shorter side.

3.3.6 *cutoff frequency*,  $n$ —the lowest frequency at which non-evanescent, dominant mode propagation can occur within a rectangular waveguide.

## 4. Summary of Test Method

4.1 A carefully machined test specimen is placed in an electromagnetic waveguide transmission line and connected to a calibrated network analyzer that is used to measure the  $S$ -parameters of the transmission line-with-specimen. A specified data-reduction algorithm is then used to calculate permittivity and permeability. If the material is nonmagnetic a different algorithm is used to calculate permittivity only. Error corrections are then applied to compensate for air gaps between the specimen and the transmission line conductor surfaces.

## 5. Significance and Use

5.1 Design calculations for radio frequency (RF), microwave and millimetre-wave components require the knowledge of values of complex permittivity and permeability at operating frequencies. This test method is useful for evaluating small experimental batch or continuous production materials used in electromagnetic applications. Use this method to determine complex permittivity only (in non-magnetic materials) or both complex permittivity and permeability simultaneously.

## 6. Interferences

6.1 The upper limits of permittivity and permeability that can be measured using this test method are restricted by the transmission line and specimen geometries, which can lead to unwanted higher order waveguide modes. In addition, excessive electromagnetic attenuation due to a high loss factor

within the test specimen can prevent determination of permittivity and permeability. No specific limits are given in this standard, but this test method is practically limited to low-to-medium values of permittivity and permeability.

6.2 The existence of air gaps between the test specimen and the transmission line introduces a negative bias into measurements of permittivity and permeability. In this test method compensation for this bias is required, and to do so requires knowledge of the air gap sizes. Air gap sizes are estimated from dimensional measurements of the specimen and the specimen holder, which can be measured with micrometers, feeler gauges, or other precision instruments. Several different error correction models have been developed, and a frequency independent series capacitor model is described in [Annex A2](#). Air gap corrections are only approximate and therefore this test method is practically limited to low-to-medium values of permittivity and permeability.

## 7. Apparatus

7.1 *Experimental Test Fixture*—The test fixture includes a specimen holder connected to a network analyzer, as shown in [Fig. 1](#).

7.2 *Network Analyzer*—The network analyzer needs a full 2-port test set that can measure transmission and reflection scattering parameters. Use a network analyzer that has a synthesized signal generator in order to ensure good frequency stability and signal purity.

7.3 *Waveguide Calibration Kit*—To define Port 1 and Port 2 measurement reference planes, calibration of the waveguide test fixture is required. A calibration kit consists of well-characterized standard devices and mathematical models of those devices. Use a through-reflect-line (TRL), an open-short-load-through (OSLT), or any other calibration kit that yields similar calibration quality to calibrate the waveguide test fixture.

### 7.4 Specimen Holder:

7.4.1 Because parameters such as specimen holder length and cross-sectional dimensions are of critical importance to the calculation of permittivity and permeability, carefully measure and characterize the physical dimensions of the specimen holder.

7.4.2 If a separate length of transmission line is used to hold the specimen, ensure that that empty length of line is also in place during calibration of the specimen holder.

7.4.3 The theoretical model used for this test method assumes that only the dominant mode of propagation exists ( $TE_{10}$  for rectangular waveguide or  $TE_{11}$  for circular waveguide). The existence of higher-order modes restricts the measurable bandwidth for a given waveguide test fixture.

7.4.4 Be sure that the specimen holder dimensions are within proper tolerances for the waveguide transmission line size in use. For an X-band rectangular waveguide transmission line the dimensions of the inner opening are denoted by “ $a$ ” the width and “ $b$ ” the height. Proper tolerances are then:

X-band waveguide width:

$$a = 22.86 \pm 0.10 \text{ mm } (0.900 \pm 0.004 \text{ in.}) \quad (3)$$

X-band waveguide height:

$$b = 10.16 \pm 0.10 \text{ mm } (0.900 \pm 0.004 \text{ in.}) \quad (4)$$

7.4.4.1 Dimensions and tolerances of other standard waveguides are in the appropriate manufacturer’s specifications and U.S. military specifications.<sup>3</sup>

## 8. Test Specimen

8.1 Make the test specimen long enough to ensure good alignment inside the holder. Also, make the test specimen long enough to ensure that the phase shift through the specimen is much greater than the phase measurement uncertainty of the network analyzer at the lowest measurement frequency. If a

<sup>3</sup> MIL-DTL-85/1F, 20 November 1998.

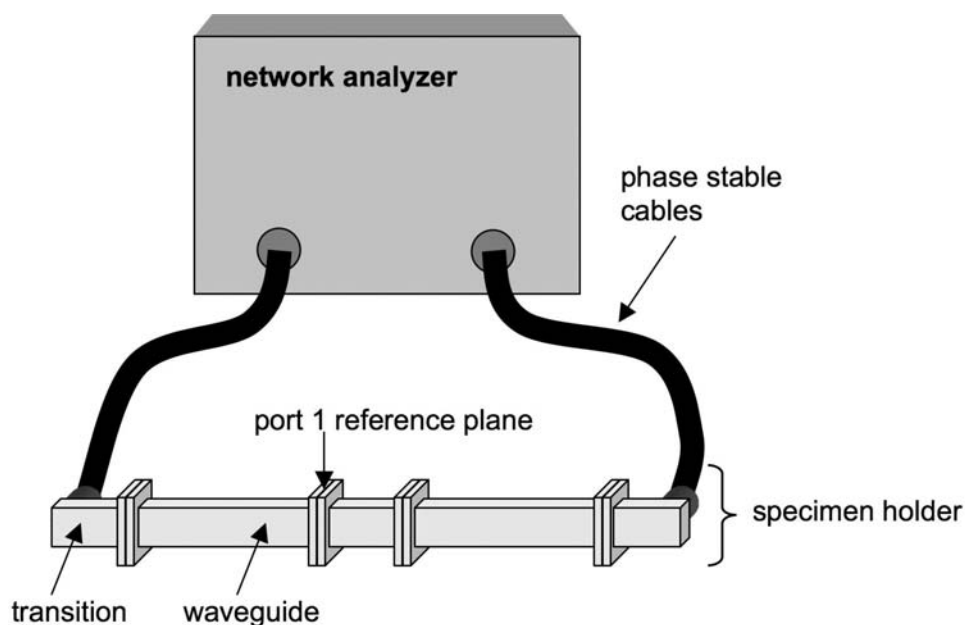


FIG. 1 Diagram of Experimental Fixture

specimen is expected to have low loss, sufficient length is also required to ensure accurate determination of the loss factor. Finally, for high loss specimens, the specimen length cannot be so long that high insertion loss prevents material property inversion.

8.2 Accurately machine the specimen so that its dimensions minimize the air gap that exists between the conductor surfaces and the specimen. In this respect, measure the specimen holder's dimensions in order to specify the tightest tolerances possible for specimen preparation. Keep physical variations of specimen dimensions as small as is practicable and include specimen dimensions and uncertainties in the report.

## 9. Preparation of Apparatus

9.1 *Inspect Network Analyzer Test Ports*—Insure that the recession of both test ports' center conductor shoulder behind the outer conductor mating plane meets the minimum specifications. Refer to network analyzer manufacturer's documentation to provide connector specifications.

9.2 *Flexing and Tightening Cables and Connectors*—Cable flexing and improperly tightened connectors introduce phase and magnitude errors into S-parameter data. For this reason, bend the test cables as little as possible, and under no circumstances bend the test cables smaller than the manufacturer's minimum recommended radius. Use of phase-stable cables is highly recommended. Insofar as possible, return the network analyzer cables to the same position during measurement of calibration standards and specimen. Use a torque wrench with the manufacturer's recommended torque to tighten connectors.

9.3 *Inspect and Clean Specimen, Specimen Holder, and Connectors*—If contamination is a concern, handle specimens with laboratory gloves. Clean specimens, connectors, and transmission lines using lint-free swabs and isopropyl alcohol (not less than 99 % pure). Blow out specimens, connectors, and transmission lines using filtered, pressurized air (not breath). Examine specimens for cracks and chips on their edges. Note any damage to specimens.

### 9.4 Network Analyzer Setup:

9.4.1 Turn on the network analyzer before beginning calibration and measurements. Refer to manufacturer documentation for minimum warm-up period.

9.4.2 Put network analyzer into step frequency mode. Set the start frequency and stop frequency as desired. Set the number of measurement points to the desired value.

9.4.3 If available, set the network analyzer's variable intermediate frequency (IF) bandwidth to less than 30 Hz, otherwise set the network analyzer data-averaging factor to a value >200.

9.4.4 Perform a two-port test fixture calibration using a waveguide calibration kit in accordance with the manufacturer's procedures and in accordance with 7.3.

## 10. Procedure

10.1 The following procedure applies to rectangular and circular waveguide.

### 10.2 Calibration Verification:

10.2.1 Insert a known verification specimen, such as crosslinked polystyrene or polytetrafluoroethylene into the waveguide so that the front surface is flush with the Port 1 reference plane.

10.2.2 Connect the waveguide flanges back together to their original configuration.

10.2.3 Make sure that the position of the waveguide feed cables are as close to the same as when the calibration was done, so that phase errors from cable movement are minimized.

10.2.4 Measure the scattering parameters of the specimen holder with verification specimen. The necessary scattering parameters depend on the calculation procedure used to determine intrinsic properties (refer to Section 11).

10.2.5 Calculate the intrinsic properties of the verification specimen from the measured scattering parameters, as described in the section on calculation. If the calculated intrinsic properties are incorrect, check the waveguide fixture to make sure connectors are properly tightened and that there is nothing wrong with the feed cables. Also check to make sure the calibration was done correctly and the inversion algorithm and phase correction were properly applied.

### 10.3 Specimen Measurement:

10.3.1 Measure and record the dimensions of the specimen with a micrometer or other precision instrument.

10.3.2 Insert the unknown specimen into the Port 1 side of the specimen holder, as shown in Fig. 2. Be careful not to damage the specimen when inserting into the holder. Make sure the front surface of the sample is flush with the Port 1 reference plane. The use of a dowel will aid in positioning the specimen so that it is flush. If the inversion algorithm to be used is not dependent on the specimen location, then special care in positioning the specimen is not necessary.

10.3.3 Record how the specimen fits (loose, snug, tight, loose at first, then tight, etc.). This subjective observation helps to determine the dimensional qualities of the specimen.

10.3.4 Connect the waveguide flanges back together to their original configuration.

10.3.5 Make sure that the position of the waveguide feed cables are identical to when the calibration was done, so that phase errors from cable movement are minimized.

10.3.6 Measure the scattering parameters of the specimen holder containing the specimen. The necessary scattering parameters depend on the calculation procedure used to determine intrinsic properties (refer to Section 11).

10.3.7 Remove specimen and if additional specimens exist, they can be measured by repeating 10.3.1 thru 10.3.6.

10.3.8 If 1 h or more has lapsed since the calibration, renew the calibration and reverify with the known verification specimen. This will account for temperature changes of the waveguide fixture, which induce phase variations.

10.3.9 Calculate the intrinsic properties of the specimen(s) from the measured scattering parameters, as described in the section on calculation.

## 11. Calculation

11.1 The selection of data reduction algorithm for calculation of material characteristics depends on several factors. Four calculations procedures are provided in this standard, as summarized in Table 1.



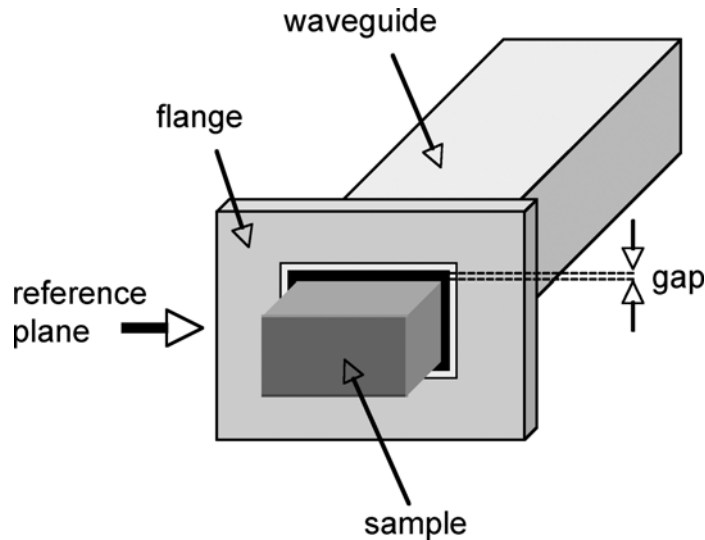


FIG. 2 Drawing of Sample Being Inserted into Waveguide

TABLE 1 Summary of Calculation Algorithms for Inverting Intrinsic Material Parameters

Inversion Algorithm Name	Required Scattering Parameters	Calculated Parameters	Comments
NRW	$S_{11}, S_{21}$	$\mu_r^*, \epsilon_r^*$	requires accurate sample positioning, no guess value needed (non-iterative)
Iterative four-parameter	$S_{11}, S_{22}, S_{21}, S_{12}$	$\mu_r^*, \epsilon_r^*$	requires initial guess values, accurate sample positioning not required
Modified NRW	$S_{11}, S_{21}$	$\epsilon_r^*$	requires accurate sample positioning, no guess value needed (non-iterative)
Iterative one-parameter	$S_{21}$	$\epsilon_r^*$	requires an initial guess value, accurate sample positioning not required

11.2 Before applying an inversion algorithm, the transmission scattering parameters ( $S_{21}$  and  $S_{12}$ ) must have a phase correction applied to account for the transmission line length displaced by the sample. For a sample of length,  $L$ , multiply the transmission scattering parameter by the factor,  $e^{-\gamma_0 L}$ , where  $\gamma_0 = \sqrt{k_c^2 - k_0^2}$ .  $k_c$  is the cutoff wavenumber ( $k_c = \pi/a$  for rectangular waveguide) and  $k_0$  is the wavenumber in air ( $= 2\pi/\lambda_0$ ).

11.3 *NRW Algorithm*—The NRW algorithm was originally developed by Nicholson and Ross<sup>4</sup> and Weir<sup>5</sup>. The intrinsic permittivity and permeability are calculated by:

$$\mu_r^* = \frac{2\pi}{\Lambda \sqrt{k_0^2 - k_c^2}} \left( \frac{1 + \Gamma}{1 - \Gamma} \right) \text{ and } \epsilon_r^* = \frac{1}{\mu_r^* k_0^2} \left( \frac{4\pi^2}{\Lambda^2} + k_c^2 \right) \quad (5)$$

$$\text{where } \Gamma = X \pm \sqrt{X^2 - 1}, X = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}, \quad (6)$$

$$T = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma} \text{ and } \frac{1}{\Lambda^2} = -\left[ \frac{1}{2\pi L} \ln(T) \right]^2 \quad (7)$$

Note that the NRW algorithm suffers from a numeric instability when the frequency corresponds to a multiple of one-half wavelength in the sample. This instability is caused in part by the large phase uncertainties, and is more likely to

happen in low-loss materials. Also note that there are other solutions to the derived equations, Eq 5-7. These other solutions can be accessed by setting  $\ln(T) = \ln(T) + j(2\pi n)$ , where  $n$  is a positive or negative integer. In most cases, the  $n = 0$  solution is correct. In addition, the ambiguity in the sign of Eq 6 can be resolved by choosing  $\text{abs}(\Gamma) \leq 1$ .

11.4 *Iterative Four-Parameter Algorithm*—The iterative algorithms begin with initial estimates at all frequencies for permittivity (and permeability). The algorithm then refines the initial estimates with an iterative technique. There are many possible solutions to the equations being used, and an initial estimate is necessary so as to select the proper root. One can determine if the initial estimates are reasonable by the stability of permittivity and permeability results. If the initial estimates start the iterative calculation on the wrong root, the calculated results tend to vary up and down and sometimes jump suddenly to another root. A set of equations relating scattering parameters and specimen length, permittivity and permeability is given as follows:<sup>6</sup>

$$F = e^{-2\gamma_0 L} S_{11} S_{22} - S_{21} S_{12} = \frac{\Gamma^2 - T^2}{1 - \Gamma^2 T^2} \quad (8)$$

$$G = (S_{21} + S_{12})/2 = \frac{T(1 - \Gamma^2)}{1 - \Gamma^2 T^2} \quad (9)$$

11.4.1 Separate the real and imaginary parts of Eq 8 and Eq 9 and take the total differential with respect to  $\epsilon_r'$ ,  $\epsilon_r''$ ,  $\mu_r'$  and

<sup>4</sup> Nicholson, A. M., and Ross, G. F., "Measurement of the Intrinsic Properties of Materials by Time Domain Techniques," *IEEE Transactions on Instrumental Measurements*, Vol IM-19, pp. 377-382, November 1970.

<sup>5</sup> Weir, W. B., "Automatic Measurement of Complex Dielectric Constant and Permeability at Microwave Frequencies," *IEEE Proceedings*, Vol 62, No. 1, pp. 33-36, January 1974.

<sup>6</sup> Baker-Jarvis, J. R., Janezic, M. D., Grosvenor, J. H., and Geyer, R. G., "Transmission/Reflection and Short-Circuit Line Methods for Measuring Permittivity and Permeability," *NIST Technical Note 1355*, May 1992.

$\mu_r''$  to generate a system of four real equations. In matrix form this looks like the following:

$$\begin{bmatrix} \frac{\partial \Re(F)}{\partial \varepsilon_r'} & \frac{\partial \Re(F)}{\partial \varepsilon_r''} & \frac{\partial \Re(F)}{\partial \mu_r'} & \frac{\partial \Re(F)}{\partial \mu_r''} \\ \frac{\partial \Im(F)}{\partial \varepsilon_r'} & \frac{\partial \Im(F)}{\partial \varepsilon_r''} & \frac{\partial \Im(F)}{\partial \mu_r'} & \frac{\partial \Im(F)}{\partial \mu_r''} \\ \frac{\partial \Re(G)}{\partial \varepsilon_r'} & \frac{\partial \Re(G)}{\partial \varepsilon_r''} & \frac{\partial \Re(G)}{\partial \mu_r'} & \frac{\partial \Re(G)}{\partial \mu_r''} \\ \frac{\partial \Im(G)}{\partial \varepsilon_r'} & \frac{\partial \Im(G)}{\partial \varepsilon_r''} & \frac{\partial \Im(G)}{\partial \mu_r'} & \frac{\partial \Im(G)}{\partial \mu_r''} \end{bmatrix} \cdot \begin{bmatrix} \Delta \varepsilon_r' \\ \Delta \varepsilon_r'' \\ \Delta \mu_r' \\ \Delta \mu_r'' \end{bmatrix} = \begin{bmatrix} \Delta \Re(F) \\ \Delta \Im(F) \\ \Delta \Re(G) \\ \Delta \Im(G) \end{bmatrix} \quad (10)$$

where the four-by-four matrix is the Jacobian of the system of four equations. In matrix notation the discretized system of equations can be represented as follows:

$$J\Delta X = Y \quad (11)$$

where we define the vector  $X$  as the estimated values of permittivity and permeability, and the vector  $\Delta X$  contains the estimated corrections for permittivity and permeability for use in the next iteration:

$$\Delta X = \begin{bmatrix} \Delta \varepsilon_r' \\ \Delta \varepsilon_r'' \\ \Delta \mu_r' \\ \Delta \mu_r'' \end{bmatrix} \quad (12)$$

If we solve for  $\Delta X$  we get:

$$\Delta X = J^{-1}Y \quad (13)$$

To solve for permittivity and permeability, start with an initial estimate of  $X$  and calculate  $\Delta X$ . The functional iteration procedure would then be:

$$X^{new} = X^{old} + J^{-1}Y \quad (14)$$

Repeat this process until  $X^{new} \approx X^{old}$ , at which point the converged values of permittivity and permeability have been found.

**11.5 Modified NRW Algorithm**—When the specimen is known to be non-magnetic, then the relative permeability can be set  $\mu_r^* = 1$ . The permittivity can then be calculated using Eq 5. Note that in this “modified” NRW algorithm, the half-wavelength instability is no longer a concern.

**11.6 Iterative One-Parameter Algorithm**—When the specimen is known to be non-magnetic and a good estimate for the permittivity is known, then permittivity can be inverted iteratively with just the  $S_{21}$  scattering parameter data. The procedure iteratively solves the following equations for permittivity:

$$S_{21} = \frac{T(1 - \Gamma^2)}{1 - \Gamma^2 T^2} \quad (15)$$

$$\text{where } \Gamma = \frac{\gamma_0 - \gamma_1}{\gamma_0 + \gamma_1} \text{ and } T = e^{-\gamma_1 L} \quad (16)$$

where  $\gamma_0 = \sqrt{k_c^2 - k_0^2}$  and  $\gamma_1 = \sqrt{k_c^2 - \varepsilon_r^* k_0^2}$ . These equations are solved in a similar manner (albeit simplified) to the four-parameter algorithm.

**11.7 Air Gap Correction**—Apply a correction for the air gap that exists between the specimen and waveguide transmission line conductor surfaces (see [Annex A2](#)).

## 12. Report

12.1 Report the following information:

- 12.1.1 Operator name, time and date of measurement,
- 12.1.2 Type of calibration (TRL, OSLT, etc),
- 12.1.3 Network analyzer setting such as start and stop frequencies, number of points, averaging factor, and type of frequency sweep,
- 12.1.4 Computational algorithm name,
- 12.1.5 Calculated values of the permittivity of the verification specimen at each measurement frequency,
- 12.1.6 Test specimen identification and origin,
- 12.1.7 Specimen dimensions and uncertainties used in data reduction,
- 12.1.8 Specimen holder identification,
- 12.1.9 Specimen holder dimensions and uncertainties used in data reduction,
- 12.1.10 Subjective description of specimen fit and estimated location of specimen in specimen holder along with estimated gap sizes between specimen and conductor surfaces, and
- 12.1.11 Calculated values of permittivity and permeability for the test specimen at each measurement frequency.

## 13. Precision and Bias

**13.1 Precision**—It is not practicable to specify the precision of the procedure in this test method for measuring permittivity and permeability because several waveguide transmission line sizes and types can be used as specimen holders for this technique. In specific applications, it is possible to estimate measurement precision by estimating the uncertainties of the measured scattering parameters and specimen dimensions, and then applying a differential analysis to the given equations.<sup>6</sup>

The sources of error in permeability and permittivity measurement include:

- 13.1.1 Errors in measuring the magnitude and phase of the scattering parameters,
- 13.1.2 Gaps between the specimen and specimen holder,
- 13.1.3 Specimen holder dimension variations,
- 13.1.4 Uncertainty in the specimen length,
- 13.1.5 Line losses and connector mismatch, and
- 13.1.6 Uncertainty in reference plane positions.

**13.2 Bias:**

13.2.1 Bias of the procedure in this test method primarily depends on proper compensation for air gaps. Existence of air gaps between the specimen and the specimen holder causes a negative bias to both permittivity and permeability calculations. Reduce this bias by applying the corrections described in [Annex A2](#). The magnitude of the air gap correction depends on the value of permittivity and permeability, and on the relative size of the air gaps.

13.2.2 When conductive paints, powders, or greases are used on the edge faces of the specimen to reduce air gaps between the waveguide transmission line conductor surfaces and the specimen, they cause measurement bias. The application of these substances will introduce a positive bias into the measurement of loss factor ( $\varepsilon_r''$ ), and could contaminate porous specimens or specimens that chemically react with the applied substance. Additionally, measurements will be greatly affected

if the applied substance is accidentally deposited on the transverse faces of the specimen.

frequency; rectangular waveguide; scattering parameter; transmission/reflection

#### 14. Keywords

14.1 dielectric constant; loss factor; magnetic permeability; measurement; microwave; millimetre-wave; permittivity; radio

## ANNEXES

### (Mandatory Information)

#### A1. LIST OF IMPORTANT SYMBOLS

$j = \sqrt{-1}$	The complex constant	$b$	Height of rectangular waveguide (m)
$c_0 = 2.9979 \times 10^8$	Speed of light in free space (m/s)	$L$	Length of specimen (m)
$\epsilon_0 = 8.854 \times 10^{-12}$	Permittivity of free space (Farads/m)	$d$	Height of specimen (m)
$\mu_0 = 4\pi \times 10^{-7}$	Permeability of free space (Henry/m)	$k_0 = \frac{2\pi}{\lambda_0}$	Wavenumber in free space (rad/m)
$f$	Measurement frequency (Hz)	$k_c = \frac{\pi}{a}$	Waveguide cutoff wavenumber (rad/m)
$\omega = 2\pi f$	Radian frequency (rad/sec)	$\gamma_0 = \sqrt{k_c^2 - k_0^2}$	Propagation constant in empty waveguide (rad/m)
$t$	Time (sec)	$\epsilon'_{rc}$	Relative real permittivity of specimen with air gap correction
$\lambda_0 = \frac{c_0}{f}$	Wavelength in free space (m)	$\epsilon''_{rc}$	Relative imaginary permittivity of specimen with air gap correction
$\epsilon_r^* = \epsilon'_r - j\epsilon''_r$	Relative complex permittivity of specimen	$\mu'_{rc}$	Relative real permeability of specimen with air gap correction
$\tan \delta = \frac{\epsilon''_r}{\epsilon'_r}$	Dielectric loss tangent of specimen	$\mu''_{rc}$	Relative imaginary permeability of specimen with air gap correction
$\mu_r^* = \mu'_r - j\mu''_r$	Relative complex permeability of specimen		
$S_{ij}$	Scattering coefficient from Port $j$ into Port $i$		
$a$	Width of rectangular waveguide (m)		

#### A2. FREQUENCY INDEPENDENT SERIES CAPACITOR MODEL FOR AIR GAP CORRECTIONS

A2.1 The air gaps between a waveguide transmission line and a specimen can be modulated by a set of capacitors in series. This results in a corrected permittivity and permeability denoted by a subscript “c.” Note that this series capacitor model assumes small gaps and low-to-moderate dielectric permittivities.

A2.2 For a sample of height,  $d$  (small transverse dimension) in a waveguide of height  $b$ , the correction for the real part of the relative permittivity is as follows:

$$\epsilon'_{rc} = \epsilon'_r \frac{d(1 + \tan^2 \delta) [b - (b-d)\epsilon'_r(1 + \tan^2 \delta)]}{[b - (b-d)\epsilon'_r(1 + \tan^2 \delta)]^2 + b^2 \tan^2 \delta} \quad (\text{A2.1})$$

A2.3 The correction for the imaginary part of the permittivity is as follows:

$$\epsilon''_{rc} = \epsilon''_r \frac{b \tan \delta}{[b - (b-d)\epsilon'_r(1 + \tan^2 \delta)]} \quad (\text{A2.2})$$

A2.4 The correction for the real part of the permeability is as follows:

$$\mu'_{rc} = \mu'_r \frac{b}{d} - \frac{b-d}{d} \quad (\text{A2.3})$$

A2.5 The correction for the imaginary part of the permeability is as follows:

$$\mu''_{rc} = \mu''_r \frac{b}{d} \quad (\text{A2.4})$$

**SUMMARY OF CHANGES**

Committee D09 has identified the location of selected changes to this test method since the last issue, D5568 – 01, that may impact the use of this test method. (Approved December 15, 2008)

(I) Revised the standard heavily throughout.

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