Measurement of Operational Amplifier Characteristics in the Frequency Domain

WILLY M. C. SANSEN, MEMBER, IEEE, MICHAEL STEYAERT, AND PAUL J. V. VANDELOO

***Abstract*—A test circuit for the automatic measurement of integrated-circuit operational amplifiers in the frequency domain has been developed. The main advantage of this test circuit over those previously reported in the literature is that it uses buffers in the feedback loop to reduce the influence of the output impedance of the operational amplifier. A fit program has been developed to extract the relevant parameters such as the transfer characteristics, the common mode rejection ratio, and the power supply rejection ratio. Examples of measurements are added for several operational amplifiers.**

I. INTRODUCTION

THE INTEGRATED-CIRCUIT operational amplifier (op-amp) is the most widely used linear circuit in the design and construction of electronic equipment. In most designs, the performance of the op-amp is assumed to be ideal. For instance, the differential-mode open-loop gain is assumed to be very high. This assumption is only reasonable at low frequencies. At high frequencies, the gain is decreased to ensure unconditional stability in the unity gain follower mode. Therefore, internal phase compensation is usually required.

Compensation critically depends on the knowledge of the op-amp transfer characteristics at frequencies around unity gain. Traditional measurement techniques of transfer functions are very often influenced by the output impedance of the op-amp. In this paper, a new measurement technique for the differential-mode open-loop gain in the high frequency domain is described, which reduces the impact of the output impedance on the measured transfer function. In addition, the measurement technique is extended to other parameters, such as common-mode and power supply rejection ratio.

A fit routine has been developed which allows the user to extract the parameters of the differential-mode open-loop gain, e.g., unity gain frequency, phase margin, low-frequency intercept, etc. First, two existing measurement methods are briefly discussed. They are used in many applications, but will be shown to give erroneous results at high frequencies. Then, the new measurement set-up is described and analyzed.

II. EXISTING TEST CIRCUITS

The first measurement circuit, which is used in many applications [1]-[4] is given in Fig. 1. The differential-mode open-loop gain is expected to be given by the ratio of Vo to $V(-)$, where $V(+)$ is considered to be at ac ground. In order to investigate the influence of the output impedance (Z), a small-

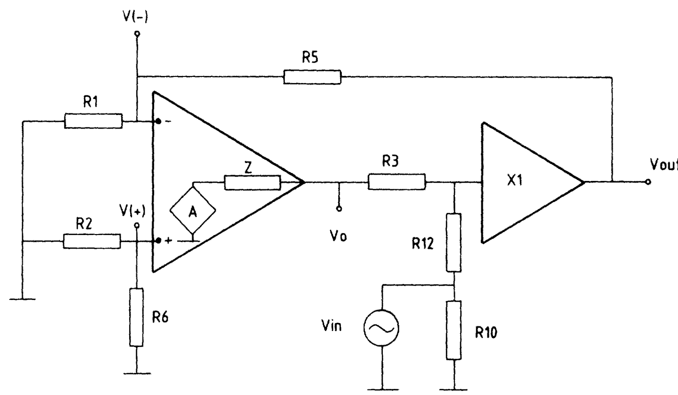

Fig. 1. Test circuit for open-loop gain measurement.

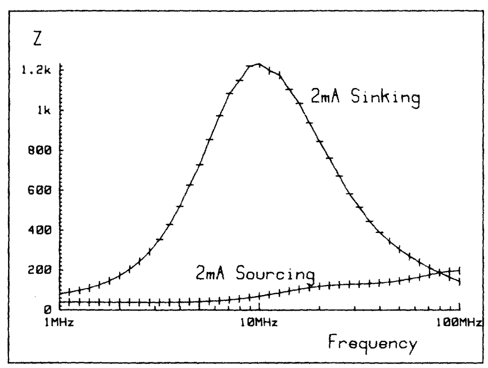

Fig. 2. Measured output impedance of an LM741.

signal equivalent circuit is used. In this analysis, the effects of the input impedance, the offset voltage, and the bias source are neglected. The transfer characteristic is then given by

$$Vo = \frac{Z \cdot Vin}{R12 + R3 + Z} - A \cdot \frac{R3 + R12}{R12 + R3 + Z} \cdot V(-) \tag{1}$$

or with $R12 \gg Z$

$$\frac{Vo}{V(-)} = -A + \frac{(R1 + R5) \cdot Z}{(R3 + Z) \cdot R1}. \tag{2}$$

At low frequencies (far below unity gain frequency), the value of A is large. Hence, the second term in (2) is negligible and the value of A is easily obtained. However, at high frequencies the output impedance (Z) of an op-amp is inductive [7], [9] as shown in Fig. 2 for an LM741 (this impedance is measured with an Hp8505/8503 and measuring S-parameters). As a result, the second term in (2) is no longer negligible. This phenomenon causes an additional bump (see Fig. 3) in the measured transfer characteristic above 1 MHz. It can be concluded that

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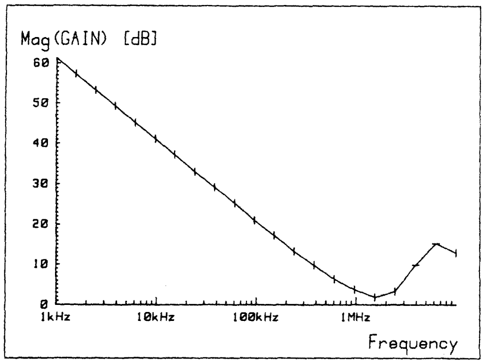

Fig. 3. Transfer function of an LM741.

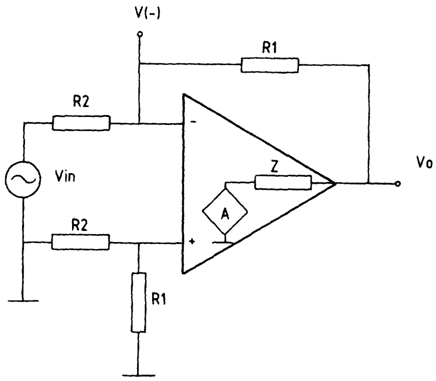

Fig. 4. Another test circuit for open-loop gain measurement.

the influence of the output impedance in the second term of (1) causes errors in the obtained value of the open-loop gain.

The second measurement circuit which is used in many applications [5], [6], is shown in Fig. 4. The Device Under Test (D.U.T.) is connected in a direct feedback loop. If Vo is measured, it is possible to calculate the differential mode open-loop gain. If again a small-signal equivalent circuit is used to analyze the influence of the output impedance (Z) of the D.U.T., the transfer function (neglecting the effects of the input impedance, offset voltage, and bias source) of the circuit is given by

$$\frac{Vo}{V(-)} = - \frac{A - Z/R1}{1 + Z/R1}. \tag{3}$$

This equation shows that the signal produced by the op-amp at high frequencies (where A is very small and Z becomes very high) can be smaller than the feedthrough of the input signal through $R1$.

The error caused by Z can be eliminated in a new measurement setup presented next.

III. The New Test Circuit for Open-Loop Measurement

The problem of the circuits used above is the finite output impedance of the D.U.T. The result is a feedthrough of the input signal through the feedback resistor. The solution for this problem is the use of an extra buffer in the feedback loop

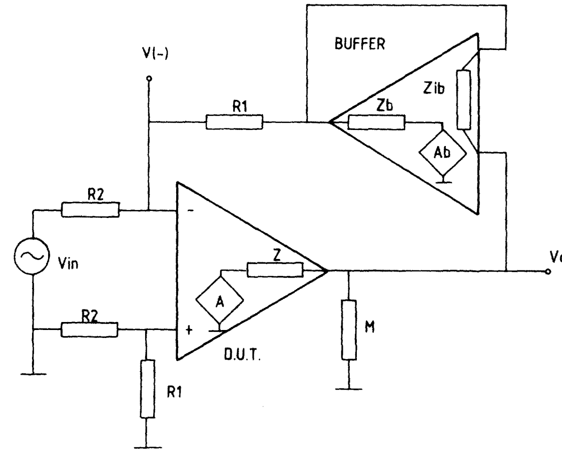

Fig. 5. New test circuit with buffer.

(Fig. 5). This extra buffer does provide the feedback signal, but the input signal can never pass through the buffer.

For the analysis of this circuit, an ac small-signal equivalent circuit of the test circuit is used. To analyze the influence of the measurement probe, an extra impedance M is added. Zb is the output impedance, Zib the input impedance, and Ab the open-loop gain of the buffer.

If we assume that (within an error of 1 percent)

1) $R1 > 100 \cdot Zb$
2) $Zib > 100 \cdot Zb$
3) $M > 100 \cdot Z$
4) $Zib > 100 \cdot Z$

the following equation can be written:

$$Vo \cdot \left(1 - \underbrace{\frac{Z}{Zib} \cdot \frac{Ab}{1 + Ab}}_{\alpha}\right) = V(-) \cdot \left(\frac{Z \cdot Zb}{Zib \cdot R1} - A + \underbrace{\frac{Zb \cdot Z}{Zib \cdot R1} \cdot \frac{Ab}{1 + Ab}}_{\beta}\right). \tag{4}$$

The maximum value of $Ab/(1 + Ab)$ is unity.

So, in the worst case with the maximum influence of alpha and beta, (4) can be reduced to

$$Vo \cdot \left(1 - \frac{Z}{Zib}\right) = V(-) \cdot \left(2 \cdot \frac{Z \cdot Zb}{Zib \cdot R1} - A\right) \tag{5}$$

or with requirement number (4)

$$\frac{Vo}{V(-)} = - A + 2 \cdot \frac{Z \cdot Zb}{Zib \cdot R1}. \tag{6}$$

if we now assume that

5) $A > 200 \cdot \dfrac{Z \cdot Zb}{Zib \cdot R1}$

the transfer function between Vo and $V(-)$ is finally given by

$$Vo = - A \cdot V(-). \tag{7}$$

The main requirement is number 3) $:M > 100 \cdot Z$. To fulfill this requirement, two extra buffers are added in the test circuit (buffer II and III in Fig. 6). Because the buffers are added in the measuring pad, it is necessary to calibrate the test circuit. The measuring software calculates the transfer function $Vo/V(-)$ out of the measured data by using the calibration data.

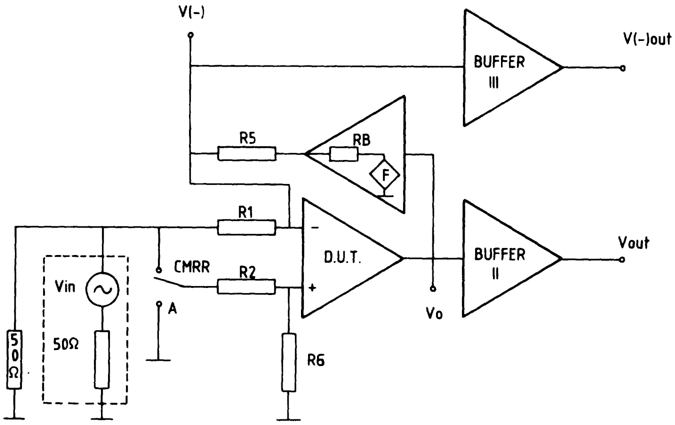

Fig. 6. Full test circuit used.

One has to keep in mind that all data are complex (real and imaginary part). Equation (7) is valid for low ($f < f$ unity gain) and high frequencies ($f > f$ unity gain), and even for a finite-gain bandwith of the buffer in the feedback loop. The finite-gain bandwith can give stability problems, but this can be solved by increasing Z/Zib, even for a D.U.T. with a negative phase margin (see measuring results in Table II).

According to requirement number 5), a minimum value for A can be found, e.g., $R1 = 10$ kΩ, $Z = Zb = 100$ Ω, $Zib = 1$ MΩ then

$$A > -70 \text{ dB}. \tag{8}$$

As a result, very small values of A can be measured above unity gain frequency. The buffer in the feedback loop must have a high input impedance ($Zib > 1$MΩ) and a very low output impedance ($Zb < 100$ Ω) at all frequencies, which is usually fulfilled by, e.g., a CA3140 in unity gain follower mode. However, the exact characteristics of the buffer are of no importance because they are calibrated out.

IV. Measurement Results

The full test circuit is shown in Fig. 6. The buffers in the test circuit are CA3140 op-amps in a unity gain follower mode: $R5 = R6 = 11$kΩ, and $R1 = R2 = 332$ Ω. The input signal is provided by a network analyzer HP3040 which is controlled by an HP 9826 desk computer. A fit routine has been developed on this computer [8] to fit the measured transfer function to the expression, by means of the least square method

$$A(j\omega) = \frac{ADMO}{(1 + j\omega/\omega 1)\,(1 + j\omega/\omega 2)\,(1 + j\omega/\omega 3)\,(1 + j\omega/\omega 4)} \tag{9}$$

where the low-frequency intercept is $ADMO$, and $\omega 1$, $\omega 2$, $\omega 3$, and $\omega 4$ are the poles of the transfer function.

In Fig. 7, the measured characteristics of a CMOS op-amp are given. As can be seen, the transfer characteristic can be easily measured over the whole frequency spectrum of the used network analyzer (10 Hz-10 MHz). The extracted parameters are given in the "Bode Approximation" (Table I). In this extraction, the -3 dB frequencies are given (F). If there is a complex pole pair, the damping and the natural frequency are given. But the most important parameters for a designer are the unity gain frequency and the phase margin (Pm). The measured and the fitted transfer function are plotted on the

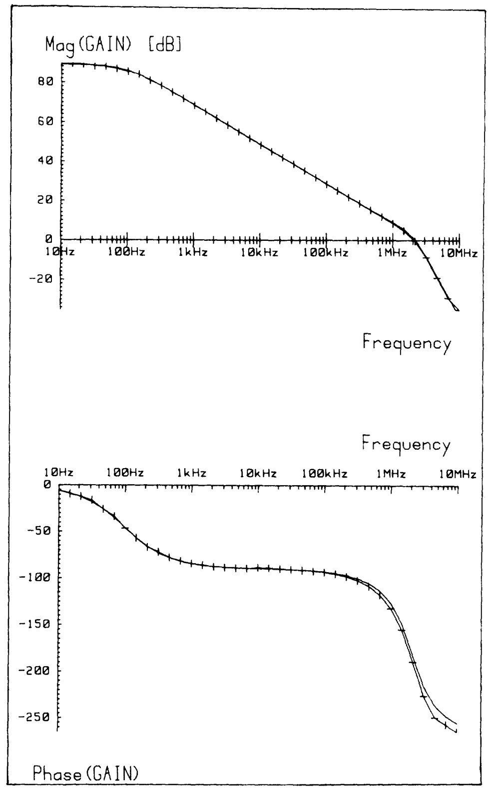

Fig. 7. Bode plot of a measured CMOS op-amp.

TABLE I

BODE APPROXIMATION:	Order = 3
Correlation=	0.99
Number of measured points=	37
Low freq intercept=	27950
ADMO*firstpole= (MHz)	2.69
Unity gain fr = (MHz)	2.10
Pm= (deg)	-7.5
F1= (Hz)	96
Complex poles:	
ωn= (rad/s)	1.31E+7
Damping= (1/(2.C))	0.586

TABLE II

BODE APPROXIMATION

Type	LM741	LM741	CMOS1(*)	CMOS2(*)	BIPOL(*)	TL081
Order	4	4	3	3	4	3
Number of points	24	24	24	37	34	25
Correlation	0.93	0.95	0.99	0.97	0.93	0.99
Low freq intercept	231400	429700	6375	125	13720	118200
ADMO*firstpole(MHz)	0.999	0.647	0.927	0.408	2.46	3.48
Unity gain fr (MHz)	0.956	0.628	0.908	0.341	1.03	3.28
Pm (deg)	67.5	69.9	66.5	55,5	-67.9	40
F1 (Hz)	4.3	1.5	145	3263	179	29
F2 (MHz)	3.2	2.1		1.0	0.70	
F3 (MHz)				1.7		
Complex poles						
ωn (rad/s)	4.03E+7	2.67E+7	2.41E+7		5.94E+6	3.98E+7
Damping	0.325	0.208	0.806		0.557	0.71

(*)= Home made

Bodeplot. As can be seen, there is almost no error—even in the phaseplot—which means that the fitting works very well. The results for a TL081 op-amp have been compared with measurements in the time domain (see Appendix A) and the error in the phase margin is less than 5 percent (Table II). Other examples of extraction parameters of measured op-amps are also given in Table II.

V. The Common Mode Rejection Ratio

The relation between the output and the input of an op-amp can be written as

$$Vo = AD(V(+) - V(-)) + AC(V(+) - V(-))/2 \tag{10}$$

where Vo is the output voltage

AD	the differential mode open-loop gain,
AC	the common-mode gain,
$V(+)$ and $V(-)$	the op-amp input signals.

The influence of the second term (common mode) has to be very small. Therefore, the common mode rejection ratio ($CMRR$), which is defined by

$$CMRR = AD/AC \tag{11}$$

has to very high.

The problems of measuring the $CMRR$ are the same as for the open-loop gain, i.e., a feedthrough of the input signal through the feedback loop. Using the same test circuit as for the open-loop gain, and measuring the transfer function $Vo/V\text{in}$ of Fig. 6, it is possible to calculate the common mode gain (see Appendix B)

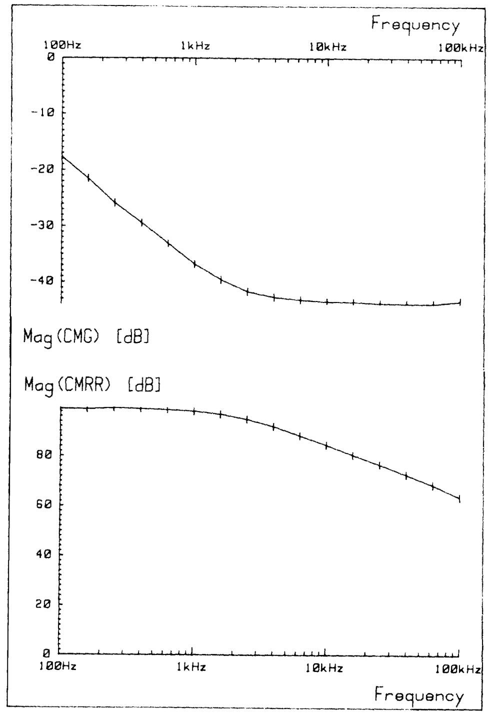

Fig. 8. Common mode gain and $CMRR$ of a measured LM741.

$$\frac{AC}{2} = \frac{\dfrac{Vo}{V\text{in}} - AD \cdot \left\{\dfrac{R6}{R6+R2} - \dfrac{R5+RB}{R5+R1+RB} - \dfrac{Vo}{V\text{in}} \cdot F \cdot \dfrac{R1}{R5+R1+RB}\right\}}{\dfrac{R6}{R6+R2} + \dfrac{R5}{R5+R1+RB} + F \cdot \dfrac{Vo}{V\text{in}} \cdot \dfrac{R1}{R5+R1+RB}} \tag{12}$$

where F represents the transfer function of the buffer, and RB its output impedance.

If we assume that (within an error of 1 percent)

1) $Vo/V\text{in} \cdot (R1 + R5) > 100 \cdot RB$
2) $F > 0.99$
3) $R1 = R2; R5 = R6$
4) $R5/(50 \cdot R1) > Vo/V\text{in}$

the equation can be reduced to

$$AC = \frac{\dfrac{Vo}{V\text{in}} \cdot \left\{1 + \dfrac{R1}{R1+R5} \cdot AD\right\}}{\dfrac{R5}{R1+R5}}. \tag{13}$$

If the value of AD is very high (at low frequencies), the value of CMRR is easily extracted from (13) and is given by

$$CMRR = \frac{R5}{R1} \cdot \left\{\frac{Vo}{V\text{in}}\right\}^{-1}. \tag{14}$$

As a result, the value of $CMRR$ is easily measured.

Requirement number 3) is a very essential one, because it is necessary to be able to neglect $R6/R6 + R2 - R5/R1 + R5$ with respect to $R1/R1 + R5 \cdot Vo/V\text{in}$. The output impedance is neglected because a buffer in the feedback loop of the D.U.T. is used, or (within an error of 1 percent)

$$\frac{R6}{R2+R6} - \frac{R5}{R1+R5} < 0.01 \frac{R1}{R1+R5} \cdot \frac{Vo}{V\text{in}} \tag{15}$$

or using (14) and replacing $R6 = R5 + \Delta R5$, $R2 = R1 + \Delta R1$

$$\frac{\Delta R5}{R5} - \frac{\Delta R1}{R1} < 0.01 \frac{R5+R1}{R1} \cdot \frac{1}{CMRR} \tag{16}$$

which means that the fractional error of $R5$, $R6$ (meaning $(R5 - R6)/R5$) and $R1$, $R2$ (meaning $(R1 - R2)/R1$) has to be smaller than the closed-loop gain divided by the $CMRR$.

On the other hand, if AD is very small (high frequencies), the value of the common mode gain is derived from (13) and given by

$$AC = \frac{Vo}{V\text{in}} \cdot \frac{R1+R5}{R1}. \tag{17}$$

By use of the desk computer, the values of AC and $CMRR$ are readily calculated with (11) and (13). For the calculation, the measurement of $Vo/V\text{in}$ and the differential mode open-loop gain is needed. In Fig. 8, the measured $CMRR$ and AC of a LM741 are given as an illustration.

V. The Power Supply Rejection Ratio

If the voltage gain of the power supply to the output of the op-amp is measured (AP), it is possible to define the power supply rejection ratio ($PSRR$) as

$$PSRR = A/AP. \tag{18}$$

Because most op-amps use a positive and negative power supply, a $PSRR$ of the positive power supply ($PSRR$ (+)) and a $PSRR$ of the negative power supply ($PSRR(-)$) can be defined. The

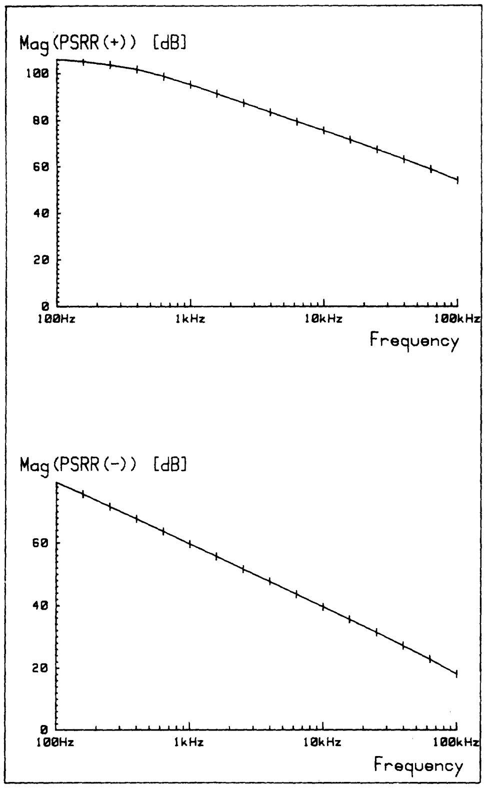

Fig. 9. *PSRR* of a measured LM741.

problems of measuring the *PSRR* are similar to those of the *CMRR* and the open-loop gain. The use of a buffer in the feedback loop again rejects the influence of the output impedance of the D.U.T. The test circuit, which is used for measuring the *PSRR* is the same as the *CMRR*, but the input signal is superimposed on the positive or negative power supply. The relation between output and input is given by

$$Vo = A(V(+) - V(-)) + AP \cdot V\text{in} \tag{19}$$

or

$$\frac{Vo}{V\text{in}} = \frac{AP}{1 + A \cdot (R1/(R1 + R5))}. \tag{20}$$

If A and $Vo/V\text{in}$ are measured, it is possible to calculate the *PSRR* and the AP with (20). In Fig. 9, an example of the *PSRR* of a LM741 is given.

VI. Conclusion

An op-amp has a finite output impedance at frequencies greater than the unity gain frequency, which causes errors in the measurement of the open-loop gain and other parameters. Therefore, it is necessary to use a buffer in the feedback loop of the device measured. In this way, it is possible to reject the influence of the feedthrough of the input signal through the feedback loop for measuring the open-loop gain over a wide frequency spectrum. The proposed test circuit is also useful for the measurement of the common mode rejection ratio and power supply rejection ratio. A program with a fit routine has been developed to automatically measure the open-loop gain and the common mode rejection ratio. The program also extracts several other parameters describing these transfer functions.

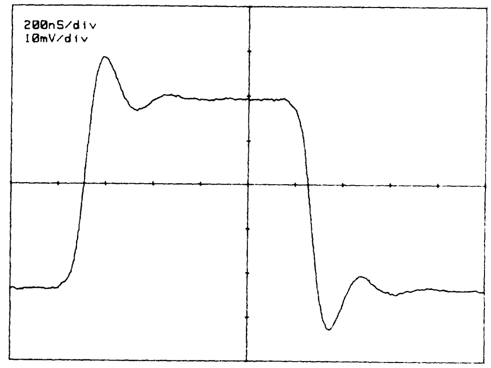

Fig. 10. Small-signal step response of a TL081.

TABLE III

TIME DOMAIN : TL081	
Step input voltage = (mV)	40.7
Overshoot = (mV)	12.4
Zeta (ξ)	0.35
Pm = (deg)	39

Appendix A
Measurement of Step Response

If a small-step input signal is applied to an op-amp in unity gain follower mode, so that the input stage is not in saturation, the response of the op-amp is that of a linear system. If the response is that of a second-order system

$$A(j\omega) = \frac{\omega n^2}{(j\omega)^2 + 2 \cdot \xi \cdot \omega n \cdot (j\omega) + \omega n^2} \tag{A1}$$

several parameters can be fitted [10] as follows:

a) the overshoot in percentage (*PO*)

$$PO = 100 \cdot \exp \frac{-\pi \cdot \xi}{\sqrt{1 - \xi^2}} \tag{A2}$$

b) the phase margin

$$Pm = \arctan \left\{ 2 \cdot \xi \cdot \left[\frac{1}{(4\xi^4 + 1)^{0.5} - 2 \cdot \xi^2} \right]^{0.5} \right\}. \tag{A3}$$

In Fig. 10, a step response of a TL081 is shown and in Table III the measured results are given. A Tektronix programmable digitizer 7921 AD is used, in combination with an HP 9826 desk computer.

Appendix B
Calculation of the Common Mode Transfer Function

In this analysis, the effects of the offset voltage and bias source of the D.U.T. and buffer, and the input imepdance of the D.U.T., are neglected. The relation between the output and the input voltages of the D.U.T. can be written as

$$Vo = AD(V(+) - V(-)) + AC(V(+) + V(-))/2 \tag{B1}$$

or

$$AC/2 = \{Vo - AD[V(+) - V(-)]\} / [V(+) + V(-)]. \tag{B2}$$

$V(+)$ is given by

$$V(+) = V\text{in} \cdot R6/(R6 + R2) \tag{B3}$$

and $V(-)$ by

$$V(-) = V\text{in} \cdot (R5 + RB)/(R5 + RB + R1) + Vo \cdot F \cdot R1/(R5 + R1 + RB) \tag{B4}$$

where F represents the transfer function of the buffer, and RB its output impedance.

Substitution of $V(+)$ and $V(-)$ in (B2) gives

$$\frac{AC}{2} = \frac{Vo - AD\left(V\text{in}\left[\dfrac{R6}{R6+R2} - \dfrac{R5+RB}{R5+RB+R1}\right] - \dfrac{R1}{R5+R1+RB}\cdot F \cdot Vo\right)}{V\text{in}\cdot\dfrac{R6}{R6+R2} + \dfrac{R5+RB}{R5+RB+R1}\cdot V\text{in} + \dfrac{R1}{R5+R1+RB}\cdot F\cdot Vo} \tag{B5}$$

or

$$\frac{AC}{2} = \frac{\dfrac{Vo}{V\text{in}} - AD\cdot\left\{\dfrac{R6}{R6+R2} - \dfrac{R5+RB}{R5+R1+RB} - \dfrac{Vo}{V\text{in}}\cdot F\cdot\dfrac{R1}{R5+R1+RB}\right\}}{\dfrac{R6}{R6+R2} + \dfrac{R5}{R5+R1+RB} + F\cdot\dfrac{Vo}{V\text{in}}\cdot\dfrac{R1}{R5+R1+RB}}. \tag{B6}$$

Acknowledgment

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References

[1] Analog Devices, *Appl. Notes*, vol. 1, pp. 21-9.
[2] Analog Devices, *Analog Circuit Design Sem.*, Apr. 1982.
[3] Burr-Brown, "Check five opamp specs in one test," *Appl. Note*, Sept. 1973.
[4] J. Okun and P. Gantt, "Use loop testing to check differential amps," *EDN*, pp. 127–130, June 9, 1983.
[5] Linear Integrated Circuits, Motorola Semiconductors, 1981–82.
[6] RCA-Linear, Integrated Circuits, 78.
[7] Gray and Meyer, *Analysis and Design of Analog Integrated Circuits.* New York: Wiley, 1977.
[8] S. Stas and M. Steyaert, "Modelleren van operationele versterkers in frequentie- en tijdsdomain", M.S. thesis, K.U. Leuven, 1983, (in Dutch).
[9] J. Solomon, "The monolithic opamp: A tutorial study," *IEEE J. Solid-State Circuits*, vol. SC-9, p. 314–332, Dec. 74.
[10] C. Chuang, "Analysis of the settling behavior of an operational amplifier," *IEEE J. Solid-State Circuits*, vol. SC-17, pp. 74–80, Feb. 1982.

Linear Least-Squares Determination of Doppler Time Derivative for NAVSPASUR-Like Signals

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***Abstract*—A method is derived for optimum estimation of doppler, doppler time derivative, and other parameters for doppler-type radar returns, using linear least-squares estimation procedures. It is used on radar returns from the Naval Space Surveillance System to obtain improvement of at least one order of magnitude in doppler measurement from previous practice; doppler derivative has been measured for the first time. The accurate measurement of doppler derivative (typically ± 0.2 Hz^2) has enabled a resolution of the inherent geometric degeneracy in the coplanar NAVSPASUR system, to provide a significant improvement in single-pass orbit determination accuracy.**

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I. Introduction

While the theoretical basis of maximum-likelihood/least-square analysis is well known, it has not been universally applied to radar parameter estimation, partially because digital sampling techniques previously have been somewhat difficult to implement. In this article, the principles of linearized least-squares analysis are applied to estimate doppler and doppler time derivative of a sample of satellite echo returns received by the Naval Space Surveillance System satellite-tracking fence. This information has been used to estimate satellite velocity and acceleration for orbital determination purposes with a pre-